Supporting Postgraduates Who Teach Mathematics and Statistics

Facilitating Problem Solving Classes

Introduction
Problem solving is one of the main activities by which we really learn mathematics. Ensuring that students get the best out of their problem classes is therefore very important, and facilitating these is one of the key tasks that postgraduates may be called upon to do. It is also one of the most difficult, because you bear the responsibility of actually encouraging the students to engage fruitfully with the activity, and for assisting them in the most effective way when they get stuck and need help.

Preparation
- **Be clear about the purpose of the session** – what is it trying to achieve, what are its learning objectives? It may be for the students to practice a new technique such as inverting matrices, a method of integration, etc.

- **Obtain the learning materials**, e.g. problem sheet set by their lecturer, well in advance of the session.

- **Make sure that you can do all the problems**. Although the odd error/hesitation on your part shows the students that you are human, too many of these and they will simply think you are incompetent, and will have little faith in you being able to help them.

- **You will need to be able to do the problems the students' way**, the way they have been taught – not necessarily your way. So you need to discuss such things with the lecturer concerned. Example: there are at least three ways to do a simple partial fractions decomposition – you need to know them all and be able to explain the connections between them.

- **You need a ‘global’ overview of a topic** to help the students with their problems and this can take several years to develop – it is not necessarily guaranteed just because you have an excellent first degree.

- **Learn as much as you can about the students** beforehand, and develop this knowledge as the course progresses. What are their
motivations and interests? What are the strengths and weaknesses of their mathematical backgrounds? Learn some names. There is no harm in a little social banter to develop rapport. Do they have any special concerns about the course?

- **Remember that the students are not necessarily like you** – their priority may not be maths, they are unlikely to be as capable as you, they might not have the same work habits as you, etc. But they are who they are, not who you want them to be and it is your job to teach them. Example: if they are engineers who only need maths as a tool, then that will influence how you approach them and their work. They may have particular difficulties – for example first years may find the transition to university teaching difficult, and they are certainly likely to be more difficult to help than say final year maths students.

- **Be prepared to adapt your personality** – maybe by nature you are a quiet retiring person who finds it difficult to develop a rapport with people. Consciously be aware of this and combat it, trying to develop ways to at least effect a more gregarious attitude – you may have to act a little (e.g. no impatience, no sarcasm). So, if necessary mentally prepare yourself before going into the classroom (like we sometimes have to when we go into an interview).

**Starting the session off**

- **Firstly, remember that you are the one in charge**, and so it is you who have to run the class. Handing out the problem sheet and expecting the students to get on with the problems, asking you for help when they need it, are not enough. You have to make it clear that you expect them to work hard, that you are anxious that they learn the material, and will go out of your way to help them in the process.

- **Commence in a business-like way**, making sure that they fully understand the purpose of the particular activity of the session. Make sure they all fully understand what they have to do.

- **Ensure that they all have the resources** to do what is required (many will not even have brought their class notes – tell them to bring them next time, and if possible alert them before the session about what they need to bring). Organise them to suit the particular activity – possibly in groups, or in alternate seats in a tiered lecture theatre, so that you can breathe down the necks of all the students!

- **Indicate some sort of schedule** – for example many students start with the first question and work progressively throughout the whole session, not getting very far in terms of overview of the topic in the time available. So you may need to say something like ‘Spend 15 minutes on the first 3 questions, then half an hour of questions 6-9, and finally make sure you spend a quarter of an hour on the last three questions’. This way, they are wrestling with a range of issues, can get your help on each, and can consolidate the details in their own time.

- **Set clear ground rules about orderly conduct of the class** from the start (see below). Your priority is to establish a conducive learning environment.
atmosphere, and most students will thank you for this, even if it means you being a little authoritarian. You might remind the students that they are paying a lot of money for you to help them and they should make sure they get their money’s worth!

**Keeping things going**

- **Keep things moving once you have started them off.** Don’t sit reading or doing your own work – for the next hour your students are the priority. And you will find that you do have to maintain momentum and push them to work to best effect. That is just human nature. You can keep reminding them that right now is the best time to sort out their difficulties, while the topic is still fresh in their minds, and while you are around to help them. It is much harder when they come to revise a few weeks before the examination – an hour spent now will save hours later.

- **Be everywhere**, move about from student to student, seeing how they are doing. Be a pest, in the nicest way, continually enquiring how they are getting on, anyone stuck, how far have you got – anyone had a look at Question 4 yet?

- **Help them with entry methods to problems**, not specific hints. Starting a problem is very difficult for some students, and they just sit staring at blank paper – don’t SHOW them how to start, because all that happens is the next step in the problem becomes a new starting point where they get stuck again. You have to show them how to ‘enter’ a problem, any problem, and how to do this for themselves (See Session 1 for further ideas here).

- **Tell them to try anything at all** – rewriting the question in their own words can help. Get them to write a list of everything they know that might be related to the problem. Have they seen anything like it before – look in their notes. Get them to discuss it with a fellow student(s).

- **Get them guessing** and assure them that this is a perfectly respectable tactic in mathematics, so if they are unsure how to start just take a guess at it and see if that leads anywhere.

- **A problem may be too hard** if the majority of the students can’t get into it. In this case invent a simpler one, or go through it on the board (see below). In any event a problem sheet should have problems gradually increasing in difficulty – sometimes called ‘step-laddering’. Few classes you teach will be master-classes, most will have a fairly wide range of ability and will need to build up their skills gradually.

- **Learn as many names as possible**, and try to build up a knowledge of their characteristics. Use any characters in the class (politely and in a friendly way of course) to help develop rapport with students. Try to bring in their interests – e.g. some might complain if working through a long list of tedious drill exercises – ask if there are any guitarists/pianists, etc in the group and ask them how they learn their chords – maths drill is the same thing. Be relaxed and friendly with them, while maintaining a respectable and professional stance.
• Make sure that everyone is engaged – don't concentrate on one particularly vocal group, gently bring in any shy isolated people, encourage them to help each other. Perhaps you might go through the first question of a particular type at one or two points in the session (but see below).

• If thinks go quiet, and you find you are not getting many questions, ask the class – ‘Anyone stuck? Anyone want to get their money’s worth? Anyone finished – do you want some more problems? etc, etc’. Keep up the pressure in a friendly, kindly way.

• Don't let them spend too long on a question. It is better to give them an idea how to proceed and then move them onto other questions because they are then experiencing a range of problems in the class, where they have the opportunity to get your help.

• Encourage students to use their notes or any other materials they have. Some students, particularly first years, find this difficult because in school the problems they do are usually very closely related to what they have just done in class. In more advanced mathematics they may have to go back quite a long way, or interpret some of their material in a different way to how it is presented in the notes.

• Get them to create their own formula sheets, etc or summaries of the session – although have your own prepared as a back up.

Working through problems on the board

• You are not really there to go through the problems for the students on the board (that would be a demonstration class), but you might sometimes have to because most people seem to be stuck at the same place.

• Using the board (black or white) effectively is quite a skill that you need to develop. It is not the sort of thing to go into here, but your staff development unit will probably be able to provide you with this training. Krantz (1999) gives a lot of useful advice on board work in the context of mathematics. Here, we are not so much interested in the technicalities of using the board, but in how you go through solutions in front of the class.

• You are simply the scribe, recounter and maybe provocateur when going through problems on the board. Make them develop the solution by asking pertinent leading questions and giving careful hints that still require them to think. When you have done it in this rough way, get them to write up the final solution tidily and carefully. When completed, leave the solution up on display and as they work through a similar problem keep asking them to refer to the solution when they get stuck again.

• Asking students to come up and work through problems on the board, is sometimes worthwhile, but only if they volunteer. No student should have to do this if they don't want to. Remember that public speaking is one of peoples’ greatest fears, and they are not there to be embarrassed but to learn.
Explaining to students

- **Explaining is one of the key arts of teaching** (Brown). In any walk of life, being able to explain things clearly and efficiently is a great gift – and particularly so for the teacher. It has intellectual components (for example, knowing the topic well enough to adapt it to your listener), and emotional components (such as not becoming impatient). All of this must be marshalled when a student asks you for help. When a student asks for help they are ideally primed to learn, so you should try to capitalise on this.

- **Never use any sort of negative, derogatory or demeaning response to a student's question.** At that particular instant your response to a student query is very important to them. If you get it wrong you can spoil the relationship with the students for the duration of the course (and other students you haven’t met yet, because your reputation will get round).

- **Be polite and helpful, but you don’t necessarily have to give the student what they are asking for** – a quick answer. ‘What is the derivative of 1/x?’ should not be answered by ‘-1/x^2’. Or by a curt ‘Look it up’. Engage them in conversation – ‘What do you need it for, have you seen it before? Show me your notes/formula sheet/book. Is it in there? Is 1/x the only way to write the reciprocal, etc, etc’. The object is to get the student to answer their own question. Don’t think this takes up too much time on one student. Almost certainly students around them will pick up and learn from it (You might notice how suddenly students nearby start to listen to you). Also, the students will see that you are on their side, that you will spend time with them, that you genuinely want to help – and this will encourage them to raise their own questions with you.

**Example**

An example might illustrate how important is the manner by which you explain things to someone. Most of us have had to learn some software in recent years. In this we are in a similar position to, say, an engineer learning mathematics. We want to be able to use it reasonably well, and will invest some effort in learning it, but it is really a small part of what we do and we have so many other things crying out for priority. If we get stuck we may ask a ‘techy’ for help. He talks way above your head, berates your lack expertise then wearily sorts it out himself. However good he is as a techy, he is no good as a teacher.

You call in techy2. He listens patiently to your problem, reflects a moment, then leans across, taps a few keys, presto done, and lets you follow suit to check you can do it yourself – job done. But is it? He has just got you to imitate him – what do you do if a similar but different thing happens again? This guy is very helpful and has solved your problem by doing it for you, but he has not helped you to learn, he is not a good teacher.

You call in techy3. She listens to your problem, probably sees the solution straight away, but doesn’t let on immediately. Instead, she asks questions to find out what you already know, how motivated you are, how much time you can devote to this task, what your precise needs are. Then she explains, using language that she now thinks you will understand, and in a depth that she thinks will benefit you most. Your problem is solved – only now your understanding will be much more permanent and portable. She is a good teacher. She appreciates that although you are willing to learn, this topic does not have a high priority, so she has to explain in the most
efficient and effective way in the context of what you already know and your possible future needs. You don’t want to be an expert, and you are not interested in the finer details – you are not lazy and stupid, you are just busy and in an unfamiliar environment. Techy3 has had to work harder to get onto your wavelength, but she has done the job required. She has also reduced the probability that you will have to go to her again!

The analogy with teaching maths to engineers for example is obvious. Being able to explain things clearly, at the level of the student, is one of the most important skills of a teacher (Brown, ‘Explaining’).

Maintaining a productive working atmosphere
Session 3 deals with encouraging student participation, and keeping students interested. That of courses applies equally to all aspects of teaching. A problem solving class is not as difficult to run smoothly as a lecture – for a start there are usually fewer students. However, it does have some special features that we will discuss here. Largely these amount to keeping the whole class on task, and curbing any disrupting or distracting behaviour.

- **Be clear about your duties, responsibilities and status** and make sure that the students are too. You may only be a year or two older than some of the students, but don’t be afraid to exert authority if necessary. If some students are ‘messing about’ and not working on the problems, get in amongst them and get them down to work. That helps them, whether they appreciate it or not, and the rest of the class.

- **Set ground rules early on and stick to them.** It is much better to be tough to begin with and ease up if necessary as the course progresses. It is much more difficult to tighten up after a lax start. Some of the ground rules you might negotiate with the students. Maybe the start time is not convenient for all of them and a change is helpful to everyone. But then stick to the new start time agreed.

- **Keep order** in the proceedings – when students (or anyone) are allowed the freedom to talk amongst themselves they can easily roam of the job in hand and things can become unruly unless some order is imposed (You might have been an unruly student yourself not long ago!). Usually humour and an appeal to the students’ good sense and courtesy will settle things down. But if it doesn’t, remain in control and politely but firmly insist that they do as they are told. If things get any worse than that consult the lecturer responsible for the course, or a member of academic staff.

- **Never be rude, sarcastic or derogatory,** no matter what the provocation. Because if you are this will alienate most of the class, and in any case it is bad manners. It is not setting a good professional example. As a young lecturer, in a moment of frustration, I sarcastically announced ‘You lot are supposed to be the brightest 10% of the population – I dread to think what becomes of the bottom 10%!’ Quick as a flash, from the back of the class, in a beautiful Irish lilt came the retort ‘They become university lecturers, sure they do!’. Fortunately that lightened things up considerably – but I was never sarcastic again with students.

- **Allow leave only for essential purposes.** Anyone leaving usually creates distractions. The class has been time-tabled and announced well in advance and
usually lasts no longer than an hour. An adult should be able to manage their
time to accommodate such things.

- **Keep on task** – that is, solving problems, or talking about solving problems. Don’t be Draconian, of course, a few minutes light banter can refresh everyone, but in the end, we do have a job to do.

- **Be aware of any diversity issues** – disabilities, multicultural and language issues. Actually, in mathematics we are relatively lucky since it is something of a universal language – however, it is well known that ‘word problems’ in mathematics can present special difficulties for foreign students for example. But be careful about stepping outside of your responsibilities and expertise on such issues, some of which can be difficult even for the experienced lecturer. Consult a member of academic staff if in doubt.

**References**


Brown *Explaining*


