RESEARCH ARTICLE

A-level Mathematics Module Choice and Subsequent Performance in First Year of an Engineering Degree

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Abstract

The A-level Mathematics qualification is based on a compulsory set of pure maths modules and a selection of applied maths modules. The flexibility in choice of applied modules has led to concerns that many students would proceed to study engineering at university with little background in mechanics. A survey of aerospace and mechanical engineering students in our university revealed that a combination of mechanics and statistics (the basic module in both) was by far the most popular choice of optional modules in A-level Mathematics, meaning that only about one-quarter of the class had studied mechanics beyond the basic module within school mathematics. Investigation of student performance in two core, first-year engineering courses, which build on a mechanics foundation, indicated that any benefits for students who studied the extra mechanics at school were small. These results give concern about the depth of understanding in mechanics gained during A-level Mathematics.

Keywords: A-level Mathematics, module choice, mechanics, engineering

Introduction

A decline, throughout the UK, in students' mathematical skills since the early 1990s has been extensively described (Hawkes & Savage 2000). More recently, concerns have been expressed regarding the structure of secondary level (A-level) mathematics and its impact on students beginning engineering degrees at university.

There are various organisations in the UK responsible for the A-level curriculum, exams and assessment. In Northern Ireland, the Council for the Curriculum, Examinations and Assessment (CCEA) provides an A-level Mathematics curriculum (CCEA 2010) which involves four core modules of pure maths (C1–C4) plus a selection of two out of four optional modules of applied maths (M1, M2, S1, S2). Table 1 summarises the content of these modules.

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Table 1 Summary of the syllabus for each of the A-level Mathematics modules provided by CCEA.

C1 Core Maths 1	Indices, quadratic equations, polynomials, graphs of functions, straight line, differentiation
C2 Core Maths 2	Circle geometry, sequences, series, binomial expansion, solving triangles, trigonometry, logs, integration for area
C3 Core Maths 3	Partial fractions, parametric equations, exponential function, further differentiation (product, quotient, chain rules), solving equations by iterative methods, numerical integration
C4 Core Maths 4	Functions, differential equations, volume of revolution, implicit and parametric differentiation, integration by substitution and parts, vectors (algebra, geometry, scalar product)
M1 Mechanics 1	Uniform acceleration, force vector, friction, equilibrium (particles and rigid bodies), moment of a force, Newton's laws of motion, impulse, momentum, conservation of linear momentum
M2 Mechanics 2	Integration and differentiation of vectors, variable acceleration, projectiles, circular motion, conical pendulum, potential and kinetic energy, work–energy principle, power
S1 Statistics 1	Presentation of data, summary measures, sample space, events, probability laws and functions, discrete and continuous probability distributions, normal distribution
S2 Statistics 2	Expectation algebra, random sampling, central limit theorem, point estimation, confidence intervals, hypothesis testing, bivariate distributions, linear regression

The modules all have the same assessment weighting and therefore the A-level Mathematics qualification comprises two-thirds pure maths and one-third applied maths. The only permitted combinations of optional modules are: M1-M2, M1-S1, S1-S2. Within the mechanics and statistics streams of modules, the level of difficulty increases sequentially. The data provided by CCEA (personal communication, 31 October 2013) demonstrates a wide variety in popularity of the three combinations (Figure 1).

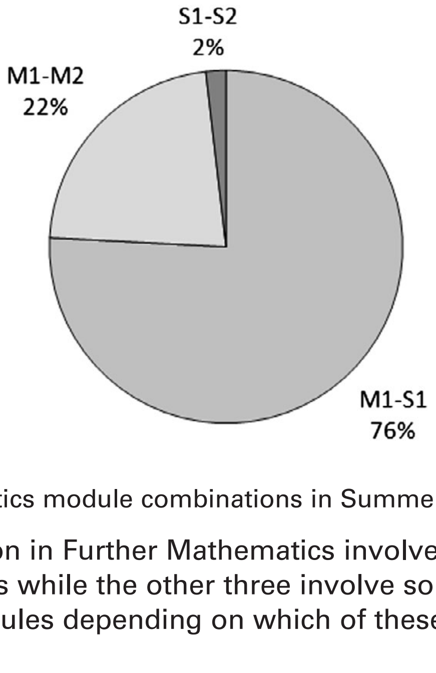

Figure 1 A-level Mathematics module combinations in Summer 2013, CCEA exam board.

A separate A-level qualification in Further Mathematics involves an additional six modules, three of which are pure maths while the other three involve some combination of mechanics and statistics modules depending on which of these have already been taken for

A-level Mathematics. Topics in further pure maths include matrices, determinants, groups, complex numbers, proof by induction and hyperbolic functions. Further mechanics topics include centre of mass, relative velocity, simple harmonic motion, satellite motion and Newton's law of restitution. There are no further statistics modules (in addition to those listed in Table 1). A-level Further Mathematics tends to be taken by only a small proportion of students in Northern Ireland, about 5% of the number taking A-level Mathematics in recent years (Joint Council for Qualifications [JCQ] 2013).

Students who follow the Edexcel exam board have even more flexibility with regard to A-level mathematics content. In addition to the four optional modules of mechanics and statistics, two further options in decision maths (D1, D2) are available (Qualifications and Curriculum Authority 2007). Topics in decision maths include linear programming, critical path analysis and flows in networks. This enables six possible combinations of optional modules: M1-M2, M1-S1, S1-S2, D1-D2, M1-D1, S1-D1.

The substantial core of pure maths in all of the A-level Mathematics curricula has been praised (Mathematics in Education and Industry [MEI] 2012). However, this report (MEI 2012), while supporting the flexibility in choice of applied maths options which allows students to tailor their qualifications to their career needs, recommends that the relative weightings of the modules be adjusted to put greater emphasis on more difficult modules such as M2, S2 and D2.

Students pursuing an engineering degree at a UK university will generally enter university directly from school where they will have studied three A-level subjects (probably including Mathematics and Physics) in their final two years. Our university selects students prior to enrolment on their degree programme by considering their A-level grades against published thresholds. Typically, a minimum of grade B will be required in Mathematics. However, given the various combinations of modules within A-level maths, described earlier, it is apparent that two students could have an A-level maths qualification, with the same grade, but have studied different topics. In an extreme example, a student could enter a mechanical engineering degree course having the required A-level maths qualification, but without having studied mechanics during A-level maths at school.

Ward-Penny *et al*. (2013) have investigated how maths teachers perceive each of the three streams of applied maths. Mechanics was considered the most difficult due to the modelling, problem solving and high algebraic content, but the most interesting. Statistics was described as the least engaging, for both students and teachers. Many respondents appreciated the values of the various applied modules to different careers and noted that certain modules would assist students' wider education. For example, studying mechanics was beneficial to A-level Physics students. The authors reported that statistics was the most commonly offered stream of applied maths; they expressed disappointment that this position has arisen due to strategic considerations with statistics seen as 'easier for the weaker students to access' (Ward-Penny *et al*. 2013, p9).

A comprehensive report (Savage & Stripp 2009) on the 'mechanics problem' detailed a significant decline in students studying mechanics at A-level since the curriculum changed in 2004 (Porkess 2003). In 2006, only 22% of A-level maths students (using the Edexcel exam board) took mechanics module M2. With a requirement to select only two applied modules, the most popular choice by schools was M1-S1 (the basic mechanics and statistics modules), believing this would maximise students' grades and expose students to a wider range of applications.

Robinson *et al*. (2005) reported the results of a survey of 242 schools across England which showed a relatively high proportion (26%) of schools offered only the basic mechanics module (M1), at most, to students studying A-level maths. A survey of first-year engineering students at three English universities showed a wide range of mechanics

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backgrounds with about 10% of students having not studied any mechanics modules within A-level maths. A study at the University of Bristol showed that 58% of engineering students in 2006 had studied M2, down from 72% in the previous year (Clements 2007).

Similar concerns exist internationally. An investigation across Europe highlighted a gap in understanding basic physics concepts, with mechanics being an important element of the gap, on transition from school to university physics (Savage & Stripp 2009). Studies in Australia identified a wide range of mechanics topics as being problematic for students (Dwight & Carew 2006, McCarthy *et al*. 2011), but students' school grades correlated only weakly with introductory mechanics results at university (McCarthy *et al*. 2011).

The current study will report the various mathematics backgrounds of the aerospace and mechanical engineering students in our university and investigate whether there was any difference in first-year performance for students with different A-level maths module combinations. Given the ongoing discussion about the structure of A-level maths, such a study is important and timely.

Methodology

The study has involved a survey of aerospace and mechanical engineering students who began their degrees at Queen's University Belfast in September 2011 and 2012 and who have therefore completed the first year. There were 52 aerospace students and 95 mechanical students in 2011/2012 while in 2012/2013 there were 46 aerospace and 120 mechanical students. The aerospace and mechanical engineering students have been treated here as a single population given that the two degree programmes have identical entry requirements and a broadly similar first-year curriculum. A-level maths module choices were surveyed by contacting the students directly.

Students' results in the first-year Solids & Structures and Engineering Dynamics modules were analysed. Solids & Structures is a first-semester course representing one-twelfth of the first-year curriculum. It covers plane frames, shear force and bending moment, torsion, stress–strain relations and beam bending stress, slope and deflection. Engineering Dynamics is a second-semester course, also weighted as one-twelfth of the first year. It covers Newton's laws of motion, conservation of energy and momentum, work–energy and impulse–momentum relationships, coefficient of restitution, moment of inertia, linear and rotary systems, variable mass problems and an introduction to mechanical vibrations. These modules provide a foundation for more advanced study in structures and dynamics in subsequent years.

The Solids & Structures coordinator commented that the A-level mechanics essential for Solids & Structures was contained within mechanics module M1. The M2 topics are not directly relevant to Solids & Structures; however, students who took M2 would be expected to be more comfortable with physics and this should benefit them in Solids & Structures. The Engineering Dynamics coordinator stated that most of the M2 topics were included in and relevant to Engineering Dynamics and that he would expect a difference in performance between students with an M2 background and those without.

Thus, it was hypothesised that any gaps in students' knowledge of mechanics, through focusing more on statistics at A-level, would have a particular effect in these important modules – hence, they were chosen for analysis. This study builds on an initial analysis which was limited to one year's class (Cole & McCartan 2013).

Student background in mathematics

Figure 2 shows the mathematics qualifications of the incoming students. A small proportion do not have A-level maths – these students have not followed the traditional path from school to university but will have studied alternative courses, perhaps at a further education college, containing some elements of maths and deemed by the university to be a suitable alternative route on to the degree programme. It is also notable that only 5% of students have taken the more advanced Further Mathematics A-level in addition to their Mathematics A-level. This is in line with the proportion of A-level Mathematics students in Northern Ireland as a whole who go on to take A-level Further Mathematics. (In England, Further Mathematics is a more significant factor with entries in A-level Further Mathematics representing 15% or 16% of entries in A-level Mathematics in each of the last six years [JCQ 2013].) Also, the AS-level Further Mathematics has not proved popular with only 2% of the class, or about four students each year, taking this qualification. It was hoped that, following the revision of the A-level Mathematics modular structure in September 2004, more students would avail of the opportunity to take Further Mathematics, at least to AS-level, in addition to A-level Mathematics in order to broaden their mathematical knowledge (Porkess 2003). (Note that AS is the first half of an A-level and can be taken as a separate qualification. Thus, while studying for three A-levels is the norm, many students take the opportunity to do an AS-level in a fourth subject.)

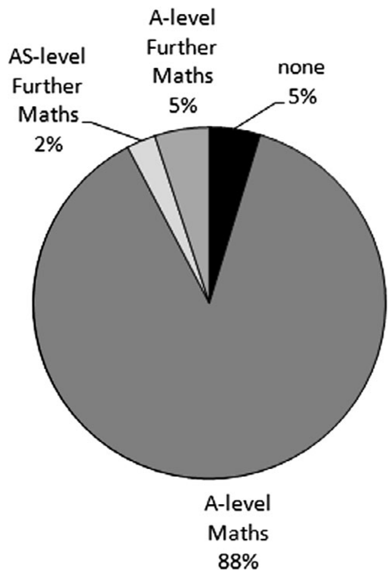

Figure 2 Students' highest qualification in A-level mathematics.

Figure 3 shows the grades attained by students in A-level Mathematics. As expected, these are mainly A*, A and B grades due to the minimum requirements for admission to a university engineering degree course.

Figure 4 shows the exam boards which provided the A-level Mathematics courses. A large majority of students in these degree courses are from Northern Ireland and many local schools use the CCEA exam board. For those not following the CCEA curriculum, the Edexcel course was the most popular alternative.

The A-level Mathematics module combinations taken by the students are shown in Figure 5 with the data for the two main exam boards presented separately. The Further Mathematics students, who would have taken a much wider range of modules, are not included here. Another small group has also been omitted, including those whose module combination is unknown and those who did not begin their aerospace or mechanical engineering degree immediately after completing their A-levels. Some students spent a year on another degree programme before deciding to transfer to engineering. They would have gained mathematical practice during that year and therefore need to be omitted from this analysis to permit investigation of how A-level Mathematics background alone influences performance in first-year engineering modules. The data in Figure 5 therefore gives a good overview of the maths background of the majority of the class (the students whose highest maths qualification is A-level Mathematics).

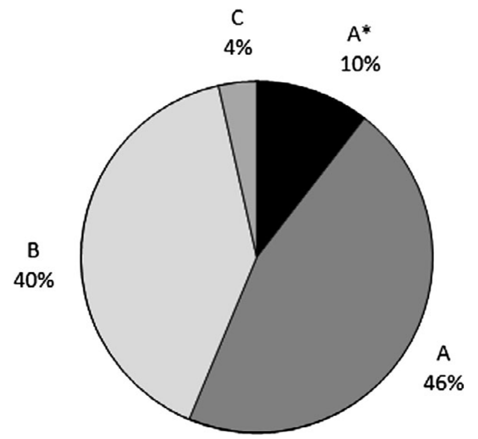

Figure 3 Students' grades in A-level Mathematics.

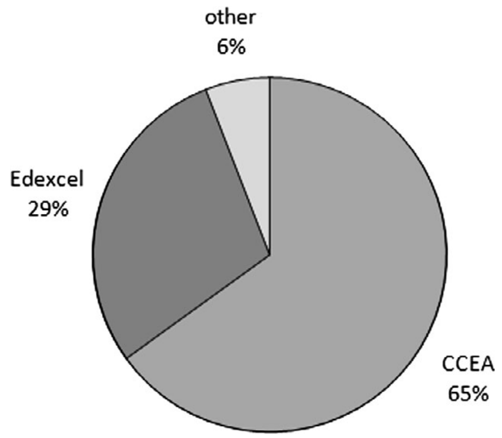

Figure 4 Exam boards from which students took A-level Mathematics.

Within the CCEA group, almost three-quarters of students have the M1-S1 background and no-one took the statistics-only combination (S1-S2). At 27%, the proportion of this engineering student group with the mechanics-only combination (M1-M2) is only slightly higher than the corresponding proportion of CCEA students overall (Figure 1). It might have been hoped that, for students going on to study engineering, the M1-M2 combination would have been more popular. That this is not the case is not surprising given that strategic concerns strongly influence which modules are provided by schools and teachers perceive statistics to be easier than mechanics (Ward-Penny *et al*. 2013). The M1-S1 choice was even more dominant for those taking the Edexcel curriculum. Within this group, no student took the statistics-only combination and no-one had a decision module as part of their A-level Mathematics. Overall, the data indicates that, for each of the 2011/2012 and 2012/2013 classes of aerospace and mechanical engineering students as a whole, about 25% of students studied mechanics beyond the basic module within school mathematics.

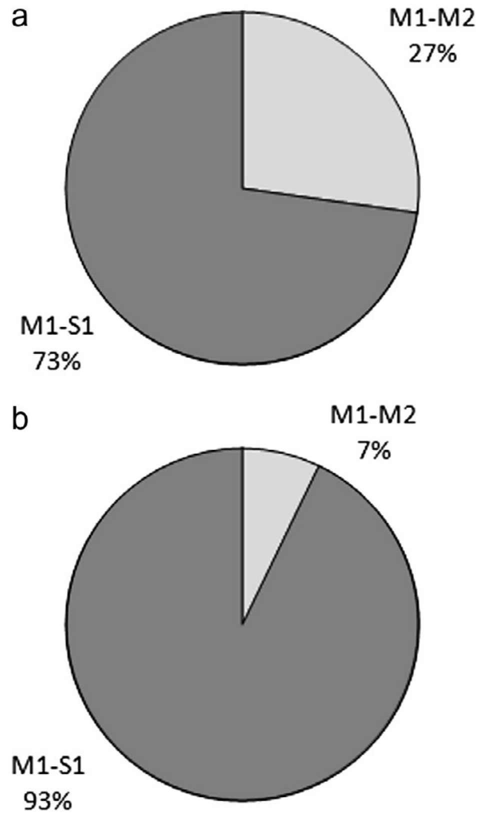

Figure 5 Module combinations for students taking A-level Mathematics from the (a) CCEA and (b) Edexcel exam boards.

Analysis of first-year performance at university

Figures 6 and 7 present individual students' results in the first-year Solids & Structures and Engineering Dynamics modules, comparing them with A-level Mathematics grades. The data is restricted to students whose maths was provided by the CCEA exam board to eliminate any effects due to differences in syllabi. Students with A-level Further Mathematics were omitted, as were those who had transferred to aerospace/mechanical engineering having previously attempted university first year elsewhere, in order to eliminate anyone with extra exposure to mechanics compared to those taking the traditional route from A-level Mathematics straight into the first year of the aerospace/mechanical degree. For the 2011/2012 class, this left a group of 70 students, 60 of whom had also studied A-level Physics. The sample was therefore reduced to those 60 students to diminish the effects of different backgrounds in physics. Of these, 15 students had done M1-M2 and 45 students had done M1-S1. The data was similarly restricted for the 2012/2013 class to leave a group of 77 students; 25 of these students had done M1-M2 and the other 52 students had done M1-S1.

Note that A-level Physics contains some of the mechanics topics covered in A-level Mathematics module M2 – in particular, projectiles, circular motion, work done, potential and kinetic energy and power. Nevertheless, it was hypothesised that the extra contact with these topics by also studying them within maths would benefit the students at university.

There is a moderate correlation with students with better A-level grades tending to gain higher marks in these modules. However, it is clear that A-level Mathematics grade is not a good predictor of performance here. For example, even those students who achieved one of the top grades in school (grade A) scored across a very wide range (approximately 30–90%) in both Solids & Structures and Engineering Dynamics.

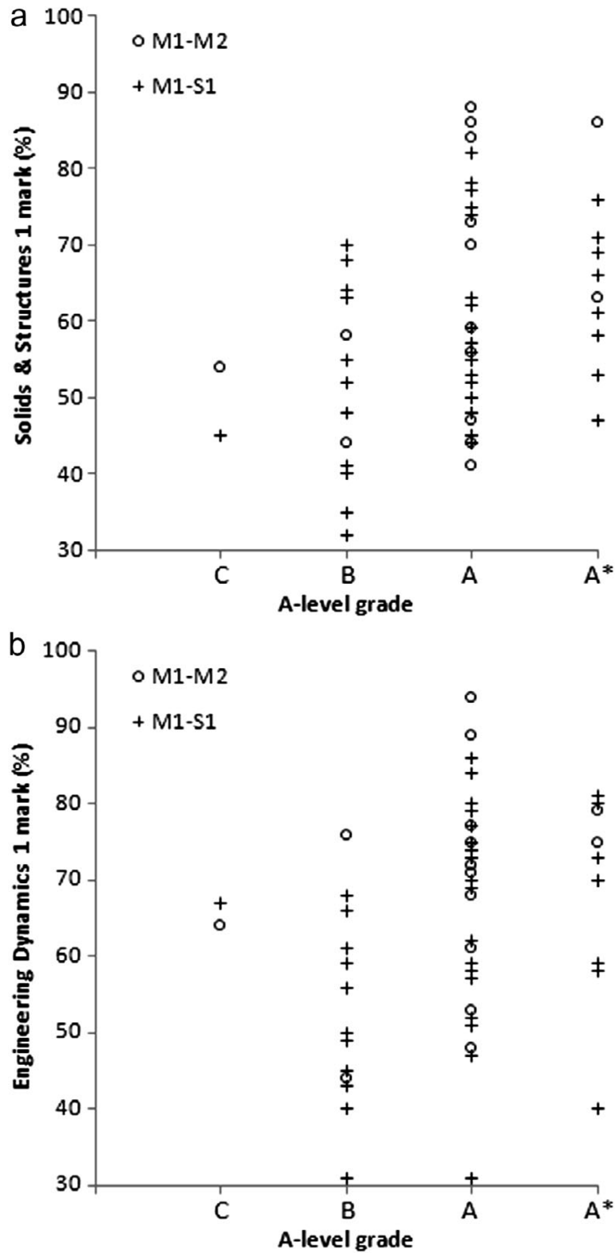

Figure 6 Comparison of students' results in first-year (a) Solids & Structures and (b) Engineering Dynamics in 2011/2012 with their A-level Mathematics grades.

Tables 2 and 3 separate the students by A-level Mathematics module combination in an attempt to identify whether this has any effect on first-year performance. One-sided *t*-tests were used to compare the mean marks achieved by students with the M1-M2 and M1-S1 backgrounds. One-sided tests were chosen since the alternative hypothesis was that the M1-M2 students would do better than the M1-S1 students given their extra experience in mechanics. In all but one case, for students with a particular A-level Mathematics grade, those who did the M1-M2 combination did better (on average) than those with the M1-S1 background. However, the differences between corresponding mean values were generally small and were significant only for the grade A students in Solids & Structures in 2011/2012 ($p < 0.1$) and the grade A* students in Engineering Dynamics in 2011/2012 ($p < 0.05$). It is noted, however, that sample sizes were small.

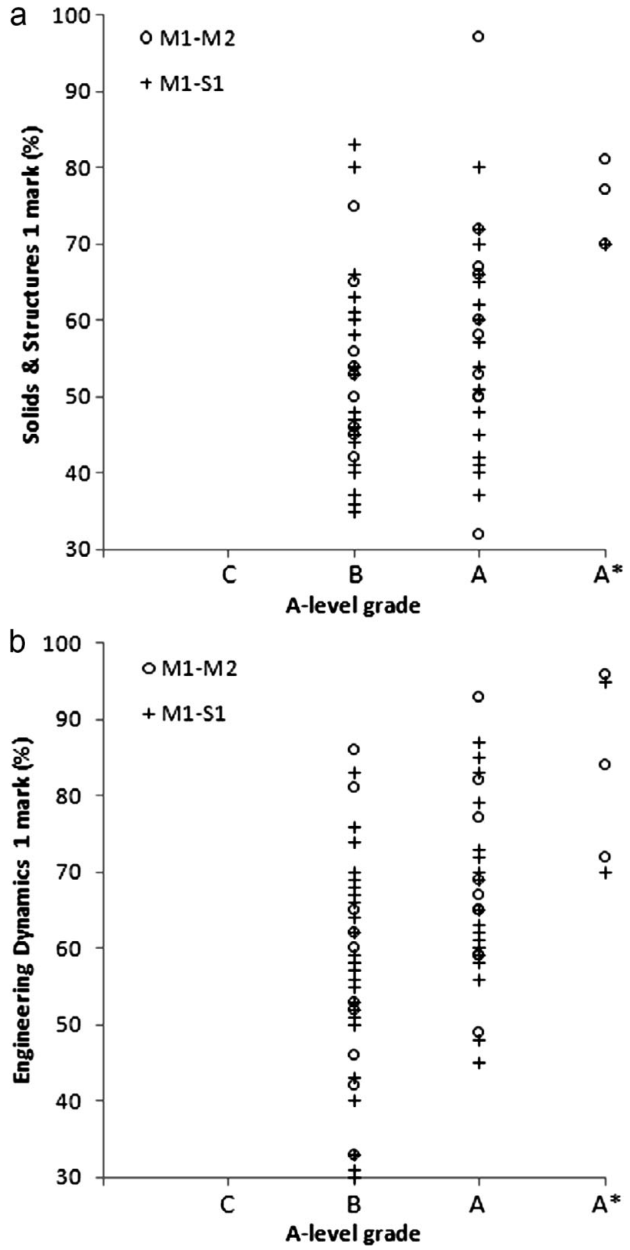

Figure 7 Comparison of students' results in first-year (a) Solids & Structures and (b) Engineering Dynamics in 2012/2013 with their A-level Mathematics grades.

More information is offered by Figures 6 and 7 which show individual students' marks in Solids & Structures and Engineering Dynamics, distinguishing the students by A-level Mathematics grade and module combination. In 2011/2012, only two of the grade A* students and two of the grade B students had an M1-M2 background. In 2012/2013, there were only five grade A* students. This limits the conclusions which can be drawn in those particular categories. When interpreting the graphs, it should also be remembered that some students may have been affected by exceptional circumstances (for example, illness) or by a lack of motivation due to, for example, realising that an engineering degree was the wrong choice for them.

For both Solids & Structures and Engineering Dynamics, there is a similar distribution of marks in the two years studied. The plots confirm that, for students with a particular A-level Mathematics grade, there is much overlap in the distributions of marks obtained by the

M1-S1 and M1-M2 students. In particular, for the grade A students in 2011/2012 and the grades A and B students in 2012/2013, while the top marks in Solids & Structures and Engineering Dynamics were usually obtained by M1-M2 students, the M1-M2 students' marks were generally spread out over a range similar to that of the M1-S1 students.

Table 2 Comparison of average marks in Solids & Structures and Engineering Dynamics in 2011/2012 with students grouped by A-level Mathematics grade and A-level Mathematics module combination.

A-level Mathematics grade	Average mark (%) A-level Mathematics module combination:		*p* Value (one-sided *t*-test)
	M1-M2	M1-S1	
Solids & Structures			
A*	74.5	62.6	0.100
A	64.8	57.3	0.083
B	51.0	51.6	0.524
Engineering Dynamics			
A*	77.0	66.4	0.036
A	70.8	66.7	0.225
B	60.0	48.3	0.187

Table 3 Comparison of average marks in Solids & Structures and Engineering Dynamics in 2012/2013 with students grouped by A-level Mathematics grade and A-level Mathematics module combination.

A-level Mathematics grade	Average mark (%) A-level Mathematics module combination:		*p* Value (one-sided *t*-test)
	M1-M2	M1-S1	
Solids & Structures			
A*	76.0	70.0	0.102
A	60.5	57.9	0.324
B	51.3	48.9	0.311
Engineering Dynamics			
A*	84.0	82.5	0.457
A	68.5	64.0	0.195
B	58.0	54.7	0.289

The Solids & Structures coordinator had stated that the M2 topics were not essential for his module but that extra exposure to physics through studying M2 would be helpful. The results presented here in Figures 6a and 7a indicate that any advantage in Solids & Structures for the M1-M2 students over the M1-S1 students was small. Indeed, even some students with an M1-S1 background, who achieved grade B in A-level Mathematics, performed strongly in Solids & Structures (marks > 70%), reaching higher marks than many with a grade A, M1-M2 background.

Given the strong relevance of the M2 topics to Engineering Dynamics, a more pronounced benefit for the M1-M2 students might have been expected in that module. However, the results in Figures 6b and 7b suggest that any advantage over the M1-S1 students was also small in Engineering Dynamics.

These findings suggest that many students with a more extensive background in mechanics have been unable to make good use of this to gain an advantage in these first-year modules at university. Thus, the research raises questions about the depth of understanding in mechanics achieved during A-level study.

Conclusions

This paper has reported the different mathematics backgrounds of students beginning aerospace and mechanical engineering degrees at our university in 2011/2012 and 2012/2013. For the great majority of students (88%), their highest mathematical qualification was A-level Mathematics while only a small proportion (5%) had achieved A-level Further Mathematics and obtaining AS-level Further Mathematics was rare (2%). A combination of mechanics and statistics was strongly the most popular choice of optional modules in A-level Mathematics, meaning that only about one-quarter of the class in both years had studied mechanics beyond the basic module within school mathematics.

A-Level Mathematics grade was not a good predictor of performance in the first-year Solids & Structures and Engineering Dynamics courses. In general, for students with a particular A-level Mathematics grade, those who did the M1-M2 combination did better (on average) than those with the M1-S1 background but the differences between corresponding mean values were usually small. The M1-M2 students' marks were generally spread out over a wide range similar to that of the M1-S1 students, suggesting that any benefits for those who had focused on mechanics (M1-M2) at school were small. Given the strong relevance of the M2 topics to Engineering Dynamics especially, these results are interesting and raise concerns about the depth of understanding in mechanics gained during A-level studies.

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