RESEARCH ARTICLE

Seven-step Problem Class Sequences: From Interactive Fun to Conceptual Learning

Alessandro Narduzzo
Physics Department, University of Bath, UK

Corresponding author:
Alessandro Narduzzo, Physics Department, University of Bath, Claverton Down, Bath BA2 7AY, UK
Email: a.narduzzo@bath.ac.uk, Phone: +44(0)1225 383324

Abstract
The introduction of technology-enabled interactive problem solving sequences involving peer and class-wide instruction in a Mathematics for scientists module is presented. While traditional chalk-and-talk is mainly used in lecturing, the combined use of an electronic audience response system and a digital pen-and-pad visualiser in problem classes leads to increased student engagement and interaction, in an informal and occasionally playful setting. This provides an opportunity for the lecturer to provide immediate targeted formative feedback that points the students to the fundamental concepts underlying the exercises.

Keywords: problem classes, classroom technologies, conceptual/procedural learning, peer instruction, zone of proximal development

Introduction
Peter Rowlett (2011) has reported how young maths researchers describe ‘a first class student’ and classifies the answers into a procedural and a conceptual category, with remembering proofs and doing well at timed exams being abilities of a procedural nature, and being able to ‘think for themselves’ and solve unseen problems falling into the conceptual category. While the value of training students to do well in both these categories can be recognised, the relative weight given to them in problem classes can sometimes be biased towards the former, especially when teaching mathematical methods or techniques. While many scientists indeed regard their mathematical knowledge as a ‘box of tools’, the mastering of such tools cannot arguably derive from proficiency in only the procedural category.

Novel classroom technologies, electronic voting systems (EVS, ‘clickers’, also audience response systems) in particular, can increase active engagement and interaction of students during contact time (see, for example, Ramesh (2011) and references cited therein). In the 1990s, physicists in the United States pioneered attempts to replace traditional lecturing with interactive engagement methods to enhance conceptual understanding (Hake 1998). The availability of the then novel technology lead to the development of what Boyle and Nicol (Boyle & Nicol 2003, Nicol & Boyle 2003,
Draper 2008) classified as Mazur and Dufresne sequences, from the first authors of the original works carried out by Mazur’s team at Harvard University (Mazur 1997, Crouch & Mazur 2001) and by the Physics Education Research Group (PERG) at the University of Massachusetts, Amherst (Dufresne et al. 1996). These activity sequences are summarised in Table 1.

Table 1 Mazur – peer-instruction – and Dufresne – class-wide discussion – sequences compared [adapted from Boyle and Nicol (2003) and Draper (2008)].

<table>
<thead>
<tr>
<th></th>
<th>Mazur sequence</th>
<th>Dufresne sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Concept question posed</td>
<td>Concept question posed</td>
</tr>
<tr>
<td>2</td>
<td>Students work individually (1–2 minutes)</td>
<td>Students work individually (1–2 minutes)</td>
</tr>
<tr>
<td>3</td>
<td>Individual answers are cast (VOTE 1)</td>
<td>Individual or group answers are cast (VOTE)</td>
</tr>
<tr>
<td>4</td>
<td>Histogram of answers displayed (FEEDBACK 1)</td>
<td>Histogram of answers displayed (FEEDBACK 1)</td>
</tr>
<tr>
<td>5</td>
<td>Small group – peer – discussions</td>
<td>Small group – peer – discussions (3–5 minutes)</td>
</tr>
<tr>
<td>6</td>
<td>Individual answers are re-cast (VOTE 2)</td>
<td>Individual answers are re-cast (VOTE 2)</td>
</tr>
<tr>
<td>7</td>
<td>Histogram of revised answers displayed (FEEDBACK 2)</td>
<td>Histogram of revised answers displayed (FEEDBACK 2)</td>
</tr>
<tr>
<td>8</td>
<td>Class-wide discussion</td>
<td>Class-wide discussion</td>
</tr>
<tr>
<td>9</td>
<td>Lecturer summarises and explains ‘correct’ answer (FEEDBACK 3)</td>
<td>Lecturer summarises and explains ‘correct’ answer (FEEDBACK 2)</td>
</tr>
</tbody>
</table>

They have three features in common: they begin with addressing a conceptual (typically multiple-choice) question, rely on the EVS to instantaneously collate all answers and anonymously display them as histograms, they end with the lecturer summarising and presenting the ‘correct’ answer. They differ in that while the Mazur sequence has an individual and a peer-instruction stage, each followed by a ‘vote and display all’ event, the Dufresne sequence consists of a peer-instruction and a class-wide discussion stage, only the first followed by the ‘vote and display all’ event. The emphasis of both these is on the use of contact time to address conceptual points that students may find challenging, while providing students in advance with materials for preparation including written notes, videos, quizzes.

Inspired by this work, a ‘seven-step’ problem class sequence has been designed, incorporating a peer-instruction component à la Mazur and a class-wide discussion à la Dufresne. This type of sequence is summarised in Table 2. The crucial difference between this activity and the Mazur and Dufresne approaches is that the starting question here is typically a procedural task, and the connections of this to fundamental concepts are explicitly explored and established after the students have attempted their own way(s) to the solution. This allows problem classes to become highly interactive environments where students not only practice, share and compare procedures and techniques, but have also got the opportunity to discuss the relevant underlying essential ideas and principles, developing and consolidating ‘deep knowledge’.

Three basic intended learning outcomes for the second year Mathematics for Scientists 4 unit (approximately 100 students) were addressed through these structured sequences: the ability to recognise some important partial differential equations and explain their features, the ability to demonstrate whether a given operator is linear or not, the ability to establish the convergence of infinite series.
Methods

The seven-step sequences were made possible by the combined use of an EVS and a digital pen-and-pad visualiser (TurningPoint Technologies and Papershow respectively; see Parmar (2012) for more on classroom technologies available at the University of Bath). The pen-and-pad could be passed around freely by students in the lecture theatre and anything written on it appeared immediately on screen and was digitally saved. A preliminary trial of this device within a smaller tutorial class was carried out with promising results, leading to the decision of large-class implementation (Narduzzo 2010).

About 20 sequences were prepared for three problem classes. Five questions could typically be addressed in each 50-minute session, the time spent on each sequence depending on their difficulty, varying from about six minutes for the easier – earlier – sequences to up to 15 minutes for the more involved ones. Three methods were used to evaluate the impact of these activities: the delivery of a targeted questionnaire to students, the collection of students’ written feedback and the comparison of exam results in learning-outcome-aligned questions with those from the previous year (when these sequences had not been introduced).

The next section presents the recorded outcomes of a few of these sequences, while the information obtained from the evaluation is presented in the discussion section.

Results

Figures 1–3 respectively show the outcomes of steps 3, 6 and 7 for a sequence designed to prove the linearity or otherwise of the given operator, using the definition of linearity. The correct answer being ‘Not a linear operator’ in this case, comparison of figures 1 and 2 indicate that the peer instruction session was indeed beneficial, taking the 55% of correct answers to 84%. After a lecturer-facilitated class-wide discussion, a student volunteered to present the correct proof to the audience; their writing could be seen by all appearing on the screen in real time and is displayed in figure 3. Figures 4 and 5 present analogous results for an easy question on the convergence of a given series. In this case a large majority identified the correct answer on their own; the peer instruction stage could therefore be kept shorter, and the same applied to the general discussion. The correct solution to this question was presented to the whole class by a student and is displayed in the top part of figure 8.

Figures 6 and 7 show the interesting case of peer instruction leading to relatively more students arriving at the incorrect answer – the series being in fact convergent: the original 48% of students getting the wrong answer becomes a 75% after peer instruction. In this case, the class-wide discussion that followed lasted longer and had input from several students; it emerged that the required re-expression of the factorials was not common.
The operator

\[ L : f(x) \rightarrow x^3 \cdot \frac{d^2}{dx^2} [f(x)] - x \]

is:

1. A linear operator
2. Not a linear operator
3. Do not know

Figure 1 Histogram from step 3 – after individual work – of a sequence aimed at testing the linearity of a given operator.

The operator

\[ L : f(x) \rightarrow x^3 \cdot \frac{d^2}{dx^2} [f(x)] - x \]

is:

1. A linear operator
2. Not a linear operator
3. Do not know

Figure 2 Histogram from step 6 – post peer instruction – of the same sequence described in figure 1.

\[
(1 \land y) \Rightarrow \gamma^2 \frac{d^2}{dx^2} [f(y)] - x
\]

\[
\text{Does } L(a \land b) = a \cdot L(\epsilon) + b \cdot L(\phi) \text{ hold?}
\]

\[
L \left( \epsilon \land b \right) = \gamma^2 \left( a \cdot \frac{dy}{dx} + b \cdot \frac{dx}{dy} \right) - x
\]

\[
L \left( \phi \land b \right) = a \left( x^2 \cdot \frac{d^2 y}{dx^2} - \phi \right) + b \left( x^2 \cdot \frac{d^2 x}{dy^2} + \phi \right)
\]

\[
= a \cdot x^2 \cdot \frac{d^2 y}{dx^2} - a \cdot \phi + b \cdot x^2 \cdot \frac{d^2 x}{dy^2} - b \cdot \phi
\]

\[
= a \cdot \left( -a \cdot \phi \right) + b \cdot \left( a \cdot \frac{dy}{dx} + b \cdot \frac{dx}{dy} \right)
\]

\[
- \chi = -a \cdot \phi
\]

Figure 3 Final outcome of step 7 – class-wide discussion – of the sequence on linearity: a student demonstrates the non-linearity of the given operator.

knowledge for the majority of the students and lead to erroneous simplifications; a brief general description of factorials and their properties was at this point given by the lecturer. A student ultimately presented the correct answer to this question, shown in the bottom
The ratio test shows that the series
\[ \sum_{n=1}^{\infty} \frac{100^n}{n!} \]

1. Is convergent;
2. Is divergent;
3. Can’t be established;
4. Do not know.

Figure 4 Outcome of step 3 of a sequence on the convergence of a given series – an easy question.

The ratio test shows that the series
\[ \sum_{n=1}^{\infty} \frac{100^n}{n!} \]

1. Is convergent;
2. Is divergent;
3. Can’t be established;
4. Do not know.

Figure 5 Outcome fully converged on the correct answer after peer instruction on the easy question on series convergence.

The ratio test shows that the series
\[ \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \]

1. Is convergent;
2. Is divergent;
3. Can’t be established;
4. Do not know.

Figure 6 Individual responses – step 3 – to a more difficult question on series convergence – the right answer being 1, convergent.
The ratio test shows that the series

\[
\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}
\]

1. Is convergent;
2. Is divergent;
3. Can't be established;
4. Do not know.

Figure 7 Responses after minimally effective peer instruction – step 6 – in the same sequence as in figure 6: while the percentage of correct answers has increased, many more students chose the incorrect answer (the series in fact converges).

Figure 8 Two students working out the correct answers to the questions of figures 4–5 (top) and 6–7 (bottom); these solutions were presented after brief (top) and extensive (bottom) class-wide discussion.

part of figure 8 and the top part of figure 9; the corrections and cancellations visible there reveal that these rearrangements and simplifications were less straightforward, with the final line added by the lecturer to show how to best present the result in an exam context. Figures 10 and 11, and the bottom of figure 9, show the outcomes of a sequence where peer instruction lead to an inversion of relative majority from incorrect to correct answer for another question on series convergence. The working presented by a student was further integrated on the blackboard by the lecturer with a general brief tutorial on the binomial expansion and its relevance to get a correct answer to this question.
Discussion

All three criteria used to probe the effectiveness of the implemented sequences were encouraging. The average marks in the two relevant exam questions increased from 81% to 84% for a basic question on identifying a partial differential equation and from 52% to 63% for the more difficult one on proving linearity of operators. A direct causal link between grade improvement and introduction of the new learning activities cannot of course be established. Typical attendance to problem classes was about 60, two thirds of the total cohort, with a similar number attending lectures. Of these students, half took part in the dedicated online questionnaire; this revealed that 67% of participants agreed or strongly agreed with the statement ‘clickers are a useful tool and their use improved my learning
experience', while 77% agreed or strongly agreed with the statement that the digital ‘pen is a useful tool and its use improved my learning experience’. The written feedback from students consisted overwhelmingly of positive comments, the recurring one being the usefulness of seeing other people's thought processes, comparing oneself to the rest of the class, and having one's work observed and commented upon. The value of peer instruction relies on learning being a process of individual construction of meaning mediated by exchange and negotiation with others, and is underpinned by Vygotsky's (1978) concepts of zone of proximal development and more knowledgeable other; in these seven-step sequences, students appear to appreciate being able to observe others at work and compare themselves with others and with the whole cohort at once. Some students also positively commented on the fun and 'difference' introduced by these activities.

The immediate feedback to the lecturer resulting from the display of EVS histograms from steps 3 and 6 of the sequences is of great value: it reveals whether the level of difficulty of a given question for a cohort is indeed as expected, and whether student interaction leads to an increase in the number of right answers. The case described in figures 6 and 7 is atypical but interesting: after individual work, only 12% got the correct answer (the series is convergent); this increased to 18% after peer instruction, while a greater number of undecided students opted for the incorrect answer 'non-convergent', this increasing from the already substantial 48% to 75% after peer instruction. The usefulness of the class-wide discussion that followed this and other similarly more difficult questions (for example, the one in figures 10 and 11) crucially relies on having established an informal and collaborative setting, in which students feel comfortable explaining what they got wrong and how. Once the students realise that the aim of the discussion is an understanding of the reasoning that went into solving a problem and a comparison of the different ways in which a question can be tackled, explaining how a certain method is totally or partially incorrect or incomplete is just as instructive as the display of the correct procedure. The social interaction – at both peer and class-wide levels – is really effective once the freedom of making mistakes is celebrated, with the lecturer explaining the positive value of such mistakes within this practice, and the students embracing this fact. This allows students to openly question and challenge one another and lecturer, ultimately leading to a large number of students contributing and receiving immediate formative feedback on their work. Addressing any related fundamental concepts at this stage is arguably more effective as

The ratio test shows that the series

\[ \sum_{n=1}^{\infty} \left( \frac{4}{5} \right)^n n^5 \]

1. Is convergent;
2. Is divergent;
3. Can't be established;
4. Do not know.

Figure 11 Peer instruction – following the outcome depicted in figure 10 – leads to the reversal of majority from the incorrect to the correct answer (convergent).
their relevance and significance can be immediately noticed. For example, by using the definition of linearity it can be established whether any given operator is linear or not; once the students have successfully learnt and practiced this, they can immediately appreciate how many definitions can generally be very powerful tools in this sense. This equips students with a wider perspective on what they learn, and contributes to building the confidence necessary to tackle unseen problems. Similarly, when evaluating limits specific ways to usefully re-express and simplify quantities can be established, and these can be used in a wide number of contexts. Explaining how the binomial expansion, useful to complete the problem displayed in figures 10 and 11, is used to find one of the fundamental limits, the value of \( e \), reminds students of the wider domain of applicability of this expansion. The class-wide discussion is the context where any fundamental ideas can be explored and consolidated (from the existence of distinct correct ways of solving the same problem to the critical assessment of any underlying assumptions, domains of applicability or ranges of validity), and common errors and misconceptions can be identified and corrected.

**Conclusion**

The seven-step sequences introduced in problem classes for part of a *Mathematics for Scientists* module resulted in sustained attendance, improved average exam grades and very positive student feedback. These sequences appear to increase student engagement and interaction, and facilitate multiple channels for targeted, immediate feedback: from students to other students and to lecturer, and from lecturer to individual – or groups of – students. Technology plays a fundamental role in helping create a collaborative learning environment where procedural training and critical enquiry can coexist and develop, and where the making, and correcting, of mistakes is encouraged.

**Acknowledgements**

This work was carried out as partial fulfilment of the Postgraduate Certificate in Academic Professional Practice for new academic staff at the University of Bath; in this context, the author is very grateful to Learning Technologist Nitin Parmar and Learning Support Officer Geraldine Jones for most valuable advice and suggestions. Enlightening conversations with Professor Harry Daniels on pedagogy and Vygotsky are also gratefully acknowledged.

**References**


