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## A brief review of **GeoGebra**: dynamic mathematics

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There have always been two strands to elementary mathematics: algebra and geometry. Therefore it is perhaps not surprising that many computer packages have specifically targeted themselves at one strand. Computer algebra systems (CAS) are focused on manipulation of *expressions* and dynamic geometry packages concentrate on *relationships* between points, lines circles and so on. Perhaps the best known examples of dynamic geometry are The Geometer's Sketchpad (<http://www.dynamicgeometry.com/>) and Cabri Geometry. (Cabri also has a rather splendid three dimensional version Cabri3D, and I have found version 2 invaluable for demonstrations in lectures.)

This article is about the package GeoGebra, the brainchild of Markus Hohenwarter, which joins geometry, algebra and calculus. On the one hand, GeoGebra is a dynamic geometry system in which you work with points, vectors, segments, lines, and conic sections. On the other hand, equations and coordinates can be entered directly. Functions can be defined algebraically and then changed dynamically afterwards. GeoGebra has a simple CAS in the background which has the ability to deal with variables for numbers, vectors and points, find derivatives and integrals of functions and offers commands like Root or Extremum. These two views are characteristic of GeoGebra: an expression in the algebra window corresponds to an object in the geometry window and vice versa. Although GeoGebra has been designed for education in secondary schools, it certainly has uses in Higher Education for demonstrations in lectures or for students to use in exploring functions, graphs and so on.

I would like to illustrate this article with some materials I have been using in my recent final year course in dynamical systems and chaos. We start by loading GeoGebra. Since GeoGebra is basically a Java applet you can do this for yourself through a web browser. In particular, go to the GeoGebra home page <http://www.geogebra.org/> and choose "web start". I encourage you to do this and work through the following simple example. For this we shall use a number of options in the "view" menu to make sure the "Axes", "Grid", "Algebra window" and "input field" are showing. We are ready to start.

In particular, I'd like to show some introductory material related to the logistic map,

$$f(x)=ax(1-x).$$

To do this we need to define  $a$ , and this is achieved by choosing to add a *slider* to the geometry window. This is done via the buttons as can be seen in the screen shot below. Clicking on the bottom right hand corner of the button itself reveals a menu of alternative, but related, buttons. The slider (see Fig 1) happens to be one of these which is not immediately visible.

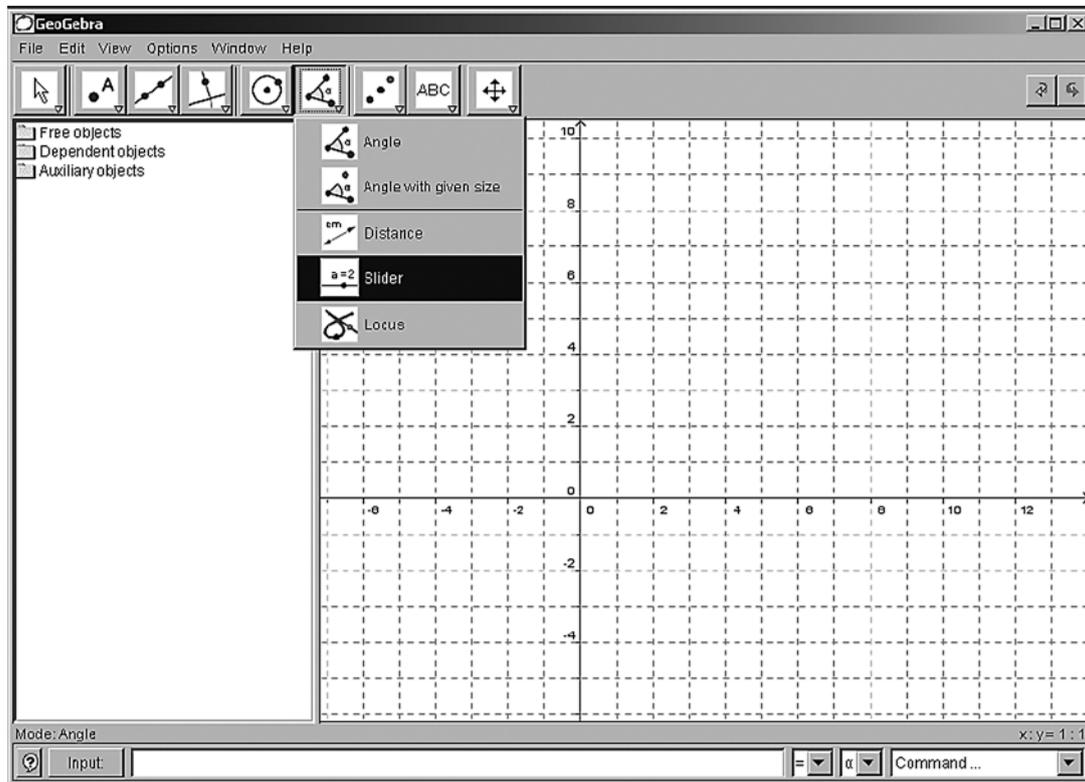


Fig 1 – Example showing Slider selection

Next the function itself is defined using the input field at the bottom of the window, literally typing in  $f(x) = a * x * (1 - x)$  in a sensible mainstream syntax. The graph is then displayed, which if  $a=0$  will be a horizontal line and so may not be apparent. Next choose the “move” tool (top left button shown above) and slide the slider left and right. This changes the value of  $a$  which dynamically changes the shape of the graph. (The properties of the slider, such as range and increment size, can be changed by right-clicking on the slider itself.)

Next choose the button to the right of the “move” tool,  $\bullet^A$  which adds points to the geometry area. In particular try adding a point on the graph. Next choose the tangent button  $\triangle$ . You may need to hunt for this under the button shown as  $\triangle$  above. A tangent is constructed by simply clicking once on the object, i.e. the graph of the function, and then on the point  $A$  previously created. An alternative to “point and click” is to use the input tool and type the syntax `tangent [ A, f ]`. The automatic command completion feature of GeoGebra is invaluable here. Everything which can be done by “point and click” can be done by such commands. Of course, now we can move both the point  $A$  and the slider  $a$  with the move tool. Notice the equation of the tangent line is shown to the left in the algebra window. So we have both an algebraically defined function and a geometrically defined point with its corresponding tangent line. This mixing of algebra and geometry is the heart of GeoGebra.

For the purposes of the Chaos course, we are most interested in  $f$  on the interval  $[0, 1]$  and the zoom tool (under  $\oplus$ ) allows this smaller region of the  $x$ -axis to

become the focus of attention. From a dynamical systems point of view fixed points, i.e. solutions of  $f(x)=x$ , are an important concept. Typing in  $y=x$  produces the straight line and the intersection of the two graphs is the solution to fixed point problem. The intersection can be found graphically with the “intersection of objects” tool  $\times$ . The stability criteria are that  $|f'(x)| < 1$  and so we have GeoGebra construct the tangent at this intersection point. The slope of the tangent line is evident from the equation as we change  $a$ . Things become a little more exciting if we look for existence of periodic orbits. These are solutions of  $f(f(x))=x$  and in general  $f^n(x)=x$  where the  $n$  denotes composition of functions. To visualize this it is straightforward to type `f(f(x))` or `f(f(f(f(x))))` into the input tool. Now move the slider  $a$  backwards and forwards to dynamically alter all the graphs at once (see Fig 2 on the following page). The effectiveness of this needs to be experienced, and is certainly visually more appealing than a sequence of static pictures such as might be provided in a book. To draw attention to one curve, right click, choose “properties” and change the line style or colour, for example.

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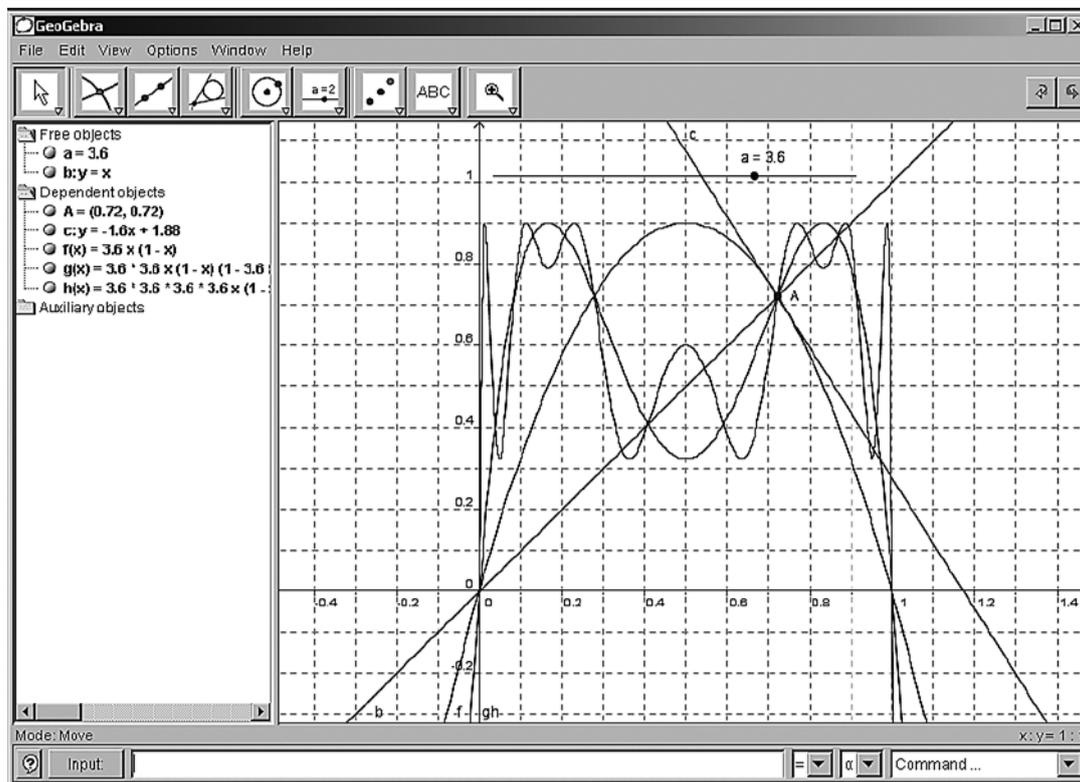


Fig 2 – Sample curves and Slider in action

What appears to be a simple school geometry and algebra tool can be very useful for simple illustrations in HE mathematics. GeoGebra is freely available to download under an open source licence (GPL) and also runs from a web browser, so that there are few barriers (technical or financial) to its use. Indeed, GeoGebra seems to be particularly easy and intuitive to learn. Files can be saved in “.ggb” format, or as dynamic web pages. If a construction is too complex to do live in a lecture a “construction steps” tool allows you to step through a pre-prepared construction without having to actually do it live. Simple, reliable and swift. GeoGebra can output files as pictures (.png) or as encapsulated postscript for publication quality illustrations.

GeoGebra is a simple piece of software, and this is its strength. It is not a fully featured CAS or “graph plotter”. Future versions promise to have many more CAS features, which will certainly make it much more useful for HE. However for now it is possible to exhaust its capabilities relatively quickly. This criticism should not put off a quick look since it is so easy to learn and use. I have found it invaluable for very quickly generating mathematically accurate diagrams for problem sheets and talks. It does very well what it sets out to achieve, and for many simple things has genuine applications in Higher Education mathematics teaching.

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The open source nature of the project has encouraged colleagues to post their examples online, with a particularly large selection of examples in German on a dedicated open website. See <http://www.geogebra.org/wiki>. These cover geometry, algebra and calculus. I would think that in many cases the days of the bespoke Java applet are numbered: GeoGebra is simply so much faster and produces flexible results.