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Frank Duckworth

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## The Duckworth/Lewis method: an exercise in Maths, Stats, OR and communications

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Frank Duckworth  
Editor of *RSS NEWS* and Statistical Consultant to the  
International Cricket Council and the England & Wales Cricket Board  
f.duckworth@rss.org.uk

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### Introduction

The Duckworth/Lewis (D/L for short) method provides a classic example of where an on-going problem in the popular arena was identified as a mathematical one and so needed a mathematical solution. The arena was the game of cricket and the idea that the solution lay in a suitable mathematical model was quite foreign to the administrators of the game. It was only the fact that several other methods were tried and yielded disastrous results that allowed D/L to be given proper consideration.

The D/L method is used to revise targets in limited-overs matches when one or both of the two sides competing have had their innings shortened after the match has started. Such interruptions usually alter the balance of the run-scoring resources available to each side and the method corrects for this imbalance.

Development of the method involved all three disciplines of maths, stats and OR, but the main discipline illustrated was that of communication - how to communicate the necessity of the mathematical approach to what was essentially a non-mathematical audience.

Fortunately, the administrators of the game of cricket, unlike those in some other popular sports, are still largely educated professionals and they include many ex-cricketers who developed their mastery of the game through the schools/college system. For instance, England and Wales Cricket Board's (ECB's) current director of cricket operations, the person who was one of the principal protagonists of the D/L method, is a graduate in economics and has studied statistics.

However, another influential section of the public, equally well-educated though not generally in mathematical disciplines, are the media commentators. From the outset they have refused to attend 'teach-in' sessions, preferring instead to perpetuate the myth that the D/L method is totally incomprehensible. Such people seem to take pride in stating that they were absolutely *useless* at maths. One reporter cynically referred to 'Vera Duckworth and Jerry Lewis', and for a long time after some sections of the media referred to it as the Vera Duckworth' method, but as it continued to prove itself, the sceptics have gradually gone quiet.

Nevertheless, after eleven years, most people involved in the game now accept that the method has stood the test of time, and even if they are unable to understand the mathematical basis they accept that it works - and works well.

Thus the D/L method can be regarded as a case study in how a mathematical approach was imposed upon a non-mathematical audience and, by the widening

appreciation of the important contribution it has made to a popular pursuit, has led to increasing awareness of the value of the mathematical sciences in areas far removed from the classroom, lecture theatre or science laboratory.

## The problem

Traditional 'first-class' cricket is a complex game for all sorts of reasons, but the numerical aspects are quite simple. Two teams play against each other and the object of each is to score as many runs as possible within the single constraint of only having 20 wickets to lose (two innings with 10 wickets to lose in each). The only other constraint is time, but if time runs out before one or the other side has been victorious the result is a draw.

In the 1960s, as a response to decreasing crowds and the desire to see matches played out to a positive conclusion in a single day, a different version of the game, known as 'one-day' or 'limited-overs' cricket was invented. In this game each team has only one innings with ten wickets to lose, but they have the additional constraint of a limited number of overs to make their score. Let us assume for this article that this is 50, though it can be different - indeed the 20 over per side game (Twenty20) is gaining in popularity year by year.

Thus a side has two separate 'run scoring resources'. As in traditional cricket, it has a limited number of wickets to lose, but now it has a limited number of overs as well. So a one-day innings is an example of a production process where maximising the output (runs) is a matter of steering a course between the limits on overs and wickets. The two constraints work in opposite directions: if one tries to optimise output relative to the overs resource (by batting more aggressively) one risks using up the wickets resource, whereas if one bats so as to preserve the wickets resource (by playing more carefully) one risks not taking the best advantage of the overs resource. As an innings progresses, the side must continually revise its strategy to steer away from the limit on either resource, and so an innings is a dynamic optimisation process.

For the side batting first, which we shall refer to throughout as 'Team 1', whose objective is to make as many runs as possible (the objective of Team 2 is slightly different as it is known how many runs are needed to win), the ideal situation is for the tenth wicket to fall on the final ball of the 50<sup>th</sup> over. In practice, many Team 1 innings end up very close to this ideal situation, but this is due entirely to the experience of the players rather than to a dynamic programming algorithm running in the pavilion with the output remotely communicated to the players on the field.

In fact, players are still learning how to optimise the score. Indications to date are that teams are still playing overcautiously in the early stages of the innings: 'to build a base from which they can accelerate later in the innings' is the commentators' standard rhetoric. Data from Twenty20 games are still rather sparse, but they do seem to suggest

that if players started a 50-over innings more as if they had only 20 overs to face, their expected total score would be considerably higher. (Here's a subject for future research.)

So much for the playing strategy. Now to the problem, which is rain.

Right from the start of the one-day game in the early sixties, it was decided that, if at all possible, all games must be played out to a result. There was no such thing as a 'draw'. But as the weather, British weather especially, is so unreliable, some way had to be found of deciding on a result if the match had to be shortened after it had started.

At first, the administrators of the game never thought this was a problem. Each side had 50 overs, so if Team 2 had their innings reduced to 25 overs due to a rain interruption, then they had half as many overs to exceed Team 1's score, so they only had to exceed half of it. And this stayed as the rule for almost the first 30 years of the game. If the match were shortened, so that the two sides had different numbers of overs available to face, the winner was the side which scored the most runs per over.

But gradually sides started complaining about the injustices that the 'average run rate' method of correction was producing. Consider the following case, for instance: Team 1 make 250 in 50 overs, Team 2 reach 126/9 in 25 overs, and the match is then abandoned. Team 2 were never going to get anywhere near the 251 they needed to win, but the rain has come to their rescue. Their run rate is slightly higher than Team 1's and so they win!

Many other similar injustices caused much concern in the game and were the subject of criticism in the media, and eventually alternative methods started to be tried out. In particular, an Australian method, called the 'most productive overs method', was adopted by the International Cricket Council (ICC), the game's governing body, for the 1992 World Cup competition. This led to a farcical situation in the semi-final match between England and South Africa, when South Africa's task of making 22 runs off the last 13 balls (generally regarded as a 50/50 situation at the time) was reduced by a shower of rain to one of making 22 runs off just one ball! ICC sent out an appeal to mathematicians of the world to come up with something better.

So at last it had been recognised that it was a mathematical problem. Enter Duckworth and Lewis.

It all began with the author writing a short discussion paper advocating an alternative method which recognised that target correction should depend not just on how many overs were lost but on the state of the innings (overs bowled, wickets down) at the time. He presented this at a 'Statistics in Sport' session of the 1992 Royal Statistical Society conference in Sheffield and a few weeks later received a letter from a maths lecturer at the University of the West of England in Bristol by the name of Tony Lewis. Tony had heard about the paper from a colleague who

had been at the Sheffield conference and decided that the collection of data and the derivation of parameters for the Duckworth formula would make a good project for one of his final year students.

To cut a long story short, the project was completed, after which Duckworth and Lewis joined forces to simplify the formula so that the method could be implemented with nothing more than a single table of numbers and a pocket calculator (the original formula required a computer program.) Having written to the Test & County Cricket Board (TCCB) offering the new method, they were invited to make presentations at Lord's, firstly to the TCCB in October 1995 and then to the ICC chief executives meeting in July 1996.

The mathematics of the method were simple, as will be seen in the next section, and could easily be understood by anyone conversant with little more than the A level maths syllabus. But it was no use using terms like 'two factor relationship', 'exponential' or 'asymptote' to the majority of the audiences at these presentations. Instead it was necessary to explain the logic by real life examples that they could identify with. Special hand-outs were prepared so they could follow the presentations, write in numbers as they were produced, and then take them back home to go through again at their leisure.

It worked! First Zimbabwe, then England and then New Zealand decided to use the D/L method during 1997, and in the following year South Africa, West Indies, India and Pakistan successively came on board. In 1999 ICC declared it to be the world standard and it has remained so ever since.

### The model

The model [1] is the simple two factor exponential relationship:

$$Z(u,w) = Z_o F(w) [1 - \exp\{-bu/F(w)\}]$$

where  $Z(u,w)$  is the average further number of runs expected to be made when there are  $u$  overs remaining and  $w$  wickets down.  $Z_o F(w)$  is the asymptotic value of further runs expected with  $w$  wickets down as  $u$  tends to infinity,  $F(0)$  being set to unity. The parameters,  $b$ ,  $Z_o$  and the nine values of  $F(w)$  were estimated from an analysis of a one-day database.

To turn the formula into something fit for consumption, the ratio  $Z(u,w)/Z(50,0)$  was calculated for all combinations of 300 values of balls remaining (six balls per over) and ten values of wickets down. These were multiplied by 100 and rounded to one decimal place, thus creating a 'Ready Reckoner' of resource percentages from which all target revisions could be calculated.

The procedure, in short, was that every time overs were lost, you used the table to read off the resource percentage remaining when play was suspended and then the resource percentage remaining when play was restarted, and hence you calculate the resource percentage lost due to the stoppage in play. You do this for each stoppage and so

obtain the total resources available to each side for their innings. Team 2's revised target is then obtained by scaling Team 1's final score according to the resources possessed by the two sides. (A slightly different adjustment used to be made, for very good reasons which will not be discussed here, when Team 2 had more resources than Team 1 and so had to be set an enhanced target.)

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At the time of introduction of the method, computers couldn't be relied upon to be present in every scoring box in the world and it was this reason that a single formula was used giving average resources, this being necessary for the method to be able to be implemented with a single table of resources. The method thus had to rely on the assumption that average performance was proportional to the mean, irrespective of the actual score. In 95 per cent of matches, this was a good enough assumption, but in the other 5 per cent, i.e. when very high scores were involved, the simple approach started to break down and consequently targets could be less equitable to the two sides.

To overcome the problem, an upgraded formula was proposed [2]:

$$Z(u,0,\lambda) = Z_o F(w) \lambda^{nF(w)+1} \{1 - \exp(-bu/[\lambda^{nF(w)} F(w)])\}$$

The additional parameter,  $\lambda$ , had to be determined for every Team 1 innings, allowing for any stoppages in that innings, and hence it had to be found by a numerical method and this could only be done by computer.

In 2003, ICC decided that computers were now ubiquitous throughout the cricket-playing world, and so they agreed to Duckworth & Lewis's suggestion to move to the upgraded formula. This gave fair targets even at the very highest scores made, but the method could no longer be implemented manually. This upgraded method, referred to as 'the Professional Edition' of D/L was introduced in October 2003 and has since been used in most competitions world-wide. The original manual method, referred to as 'the Standard Edition' of D/L, has been retained for use at lower levels of the game where computers cannot be guaranteed to be available in every score box.

## In retrospect

To date the method has been used in over 900 top flight games worldwide in over 25 countries. As well as rain, it has been used due to floodlight failure, crowd riots, a sandstorm, a snow storm and even the sun!

An important factor in getting a mathematical method adopted was the backlash from the disastrous effects of the method used in the World Cup of 1992. Nevertheless, the game's administrators are still not keen to adopt mathematical approaches when the consequences are less visible. For instance, they still prefer to stick with 'net run rate' as a second criterion for splitting sides in leagues or min-leagues, despite the D/L based alternatives that are available.

And nowhere is the lack of mathematical logic better illustrated than in the result of a match won by Team 2. As a hang-over from the traditional game of cricket, the result is expressed in terms of the number of wickets they had in hand when they reached their target. So if Team 2 win by hitting the last ball of the 50<sup>th</sup> over for four runs, ... i.e. the result could have gone either way right up to that last ball, ... if they had only lost say 4 wickets, the result would have been 'Team 2 won by 6 wickets'. A simple equivalent runs margin is produced for every game, but they don't quote it.

A similar lack of use of the maths of D/L is to be found in the ICC's one-day player ratings, whereby if a batsman gets out going for a suicidal run off the last ball of the 50<sup>th</sup> over, which logically he should always do, his rating suffers. But that's another battle to be fought.

## References

1. Duckworth, F.C. and Lewis, A.J. (1998), A fair method of resetting the target in interrupted one-day cricket matches. *Journal of the Operational Research Society*, Vol. 49 No. 3, pp. 220-227.
2. Duckworth, F.C. and Lewis A.J. (2004), A successful Operational Research intervention in one-day cricket. *Journal of the Operational Research Society*, Vol. 55, No 7, pp. 749-759.

<sup>1</sup> The author was aware that this lady was a fictitious character in a TV soap opera about a street in the Manchester area, even though he had never exposed himself to a single episode; however, when interviewed about VERA he now replies that it is a very appropriate acronym (Very Equitable Rain Adjustment), which usually has the effect of ending that line of questioning.